Logics For Epistemic Programs
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Outline

Introduction

We construct logical languages which allow one to represent a variety of possible types of changes affecting the information states of agents in a multi-agent setting.We formalize these changes by defining a notion of *epistemic program*.

THESIS I. Let *s* be a social situation involving the intuitive concepts of knowledge, justifiable beliefs and common knowledge among a group of agents.

Then we may associate to *s* a mathematical model **S**. (**S** is a multi-agent Kripke model; we call these *epistemic state models*.) The point of the association is that all intuitive judgements concerning *s* correspond to formal assertions concerning **S**, and vice-versa.

THESIS II Let σ be a *social "action"* involving and affecting the knowledge (beliefs, common knowledge) of agents.

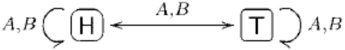
This naturally induces a *change of situation*; i.e., an operation o taking situations s into situations o(s). Assume that o is presented by assertions concerning knowledge, beliefs and common knowledge facts about s and o(s), and that o is completely determined by these assertions. Then

(a) We may associate to the action σ a mathematical model Σ which we call an *epistemic action model*. (Σ is also a multi-agent Kripke model.) The point again is that all the intuitive features of, and judgments about σ correspond to formal properties of Σ.

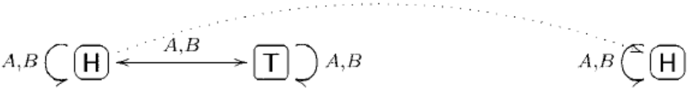
(b) There is an operation $\otimes$ taking a state model **O** and an action model Σ and returning a new state model $\mathbf{S}\otimes\Sigma$. So each Σ induces an *update operation* O on state models: $O(\mathbf{S}) = \mathbf{S}\otimes\Sigma$.

Now we have some scenarios:

SCENARIO 1. *The Concealed Coin.*



SCENARIO 2. *The Coin Revealed to Show Heads.*

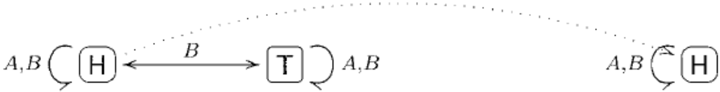


SCENARIO 2.1. *The Coin Revealed to Show Tails.*

SCENARIO 2.2. *The Coin Revealed.*

SCENARIO 3. *A Semi-private Viewing of Heads*

SCENARIO 3.1. *B's Turn*

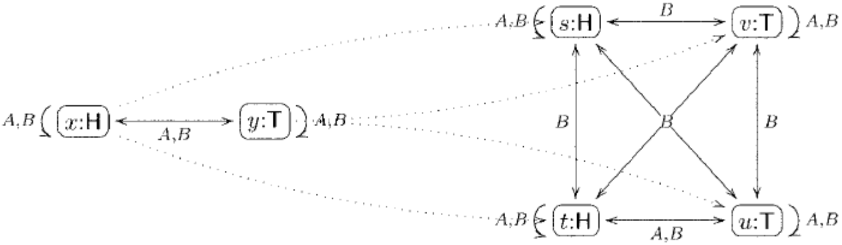


SCENARIO 4. *Cheating*

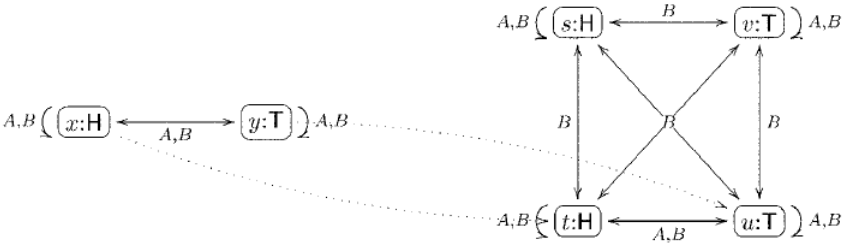
SCENARIO 5. *More Cheating*

SCENARIO 6. *Lying*

SCENARIO 7. *Pick a Card*



SCENARIO 8. *Common Knowledge of (Unfounded) Suspicion*



SCENARIO 8.1. *Private Communication about the Other*

2.1. *State Models and Epistemic Propositions*

A *state model* is a triple $\mathbf{S} = (S, \stackrel{\mathcal{A}}{\to}_{\mathbf{s}}, \| \cdot \|_{\mathbf{S}})$. $\| \cdot \|_{\mathbf{S}}$: *ATSen* $\to \mathcal{P}(S)$.

DEFINITION. Let StateModels be the collection of all state models. An *epistemic proposition* is an operation φ defined on StateModels such that for all S $\in$ StateModels, $\varphi_{\mathbf{S}} \subseteq$ S.

The collection of epistemic propositions is closed in various ways.

1. For each atomic sentence p we have an atomic proposition **p** with $\mathbf{p}_s = \| p \|_s$.
2. If φ is an epistemic proposition, then so is $\neg\varphi$, where $(\neg\varphi)_s = S \backslash \varphi_s$.
3. If C is a set or class of epistemic propositions, then so is $\bigwedge C$,with
$$(\bigwedge C)_s = \bigcap \{\varphi_s : \varphi \in C\}$$
4. Taking C above to be empty, we have an *"always true"* epistemic proposition **tr**, with $\mathbf{tr}_s = S$.
5. We also may take C in part (3) to be a two-element set $\{\varphi, \psi\}$; here we write $\varphi \wedge \psi$ instead of $\bigwedge\{\varphi, \psi\}$.We see that if φ and ψ are epistemic propositions, then so is $\varphi \wedge \psi$, with $(\varphi \wedge \psi)_s = \varphi_s \cap \psi_s$.

6. If φ is an epistemic proposition and $A \in \mathcal{A}$, then $\Box_A\varphi$ is an epistemic proposition, with

$$(2) \qquad (\Box_A\varphi)_s = \{s \in S : \textit{if} \quad s \stackrel{A}{\to} t, \textit{then} \quad t \in \varphi_s\}$$

7. If φ is an epistemic proposition and $\mathcal{B} \subseteq \mathcal{A}$,then $\Box^*_{\mathcal{B}}$ is an epistemic proposition, with

$$(\Box^*_{\mathcal{B}}\varphi)_s = \{s \in S : \textit{if} \quad s \stackrel{\mathcal{B}^*}{\to} t, \textit{then} \quad t \in \varphi_s\}$$

Here $s \stackrel{\mathcal{B}^*}{\to} t$ iff there is a sequence

$$\text{s}=u_0 \stackrel{A_0}{\to} u_1 \stackrel{A_1}{\to} \cdots \stackrel{A_n}{\to} u_{n+1}=\text{t}$$

Take senario 3 as an example here:

English	Formal rendering	Semantics
the coin shows heads	H	$\{t\}$
A knows the coin shows heads	$\Box_A$H	$\{s\}$
A knows the coin shows tails	$\Box_A$T	$\{t\}$
B knows that the coin shows head	$\Box_B$H	$\emptyset$
A knows that B doesn't know it's heads	$\Box_A \neg \Box_B$H	$\{s, t\}$
B knows that A knows that B doesn't know it's heads	$\Box_B \Box_A \neg \Box_B$H	$\{s, t\}$
it is common knowledge that either A knows it's heads or A knows that it's tails	$\Box^*_{A,B}(\Box_A \mathsf{H} \vee \Box_A \mathsf{T})$	$\{s, t\}$
it is common knowledge that B doesn't know the state of the coin	$\Box^*_{A,B} \neg(\Box_B \mathsf{H} \vee \Box_B \mathsf{T})$	$\{s, t\}$

An *update* **r** is a pair of operations

$$\mathbf{r} = (\mathbf{S} \mapsto \mathbf{S(r)}, \mathbf{S} \mapsto \mathbf{r_S})$$

where for each $\mathbf{S} \in$ *StateModels*, $\mathbf{r_S} : \mathbf{S} \to \mathbf{S(r)}$ is a transition relation. We call $\mathbf{S} \mapsto \mathbf{S(r)}$ the *update map*, and $\mathbf{S} \mapsto \mathbf{r_S}$ the *update relation*.

Examples: Pub φ, ?φ

The collection of updates is closed in various ways.

1. *Skip*: there is an update **1** with **S(1)**=**S**,and 1_S is the identity relation on **S**.
2. *Sequential Composition*: if **r** and **s** are epistemic updates, then their composition **r; s** is again an epistemic update, where $\mathbf{S(r;\,s)} = \mathbf{S(r)(s)}$, and $\mathbf{r;\,s_S} = \mathbf{r_S}; \mathbf{s_{S(r)}}$.

3. *Disjoint Union(or Non-deterministic choice)*: The set of states of the model $\bigsqcup_X \mathbf{r}$ is the disjoint union of all the sets of states in each model **S(r)**:

   $\{(s, \mathbf{r}) : \mathbf{r} \in X \text{ and } s \in \mathbf{S(r)}\}$

   (t,**r**)$\xrightarrow{A}$(u,**s**) iff if **r**=**s** and t$\xrightarrow{A}$u in **S(r).**

   $\| p \| = \{(s, \mathbf{r}) : \mathbf{r} \in X \quad \textit{and} \quad s \in \| p \|_{\mathbf{S(r)}} \}$

   t $(\bigsqcup_X \mathbf{r})_S$(u,s) iff t$s_S$ u

4. Special case: $\mathbf{r} \sqcup \mathbf{s} = \bigsqcup\{\mathbf{r}, \mathbf{s}\}$

5. Another special case: *Kleene star (iteration)*.

$$r^{*}{=}\bigsqcup\{\ \mathbf{1},\mathbf{r},\mathbf{r}\cdot\mathbf{r},\cdots,\ r^{n}\ \cdots\ \}$$

   where $\mathbf{r}^{n}$ is is recursively defined by $\mathbf{r}^{0}$,$\mathbf{r}^{n+1}$=$\mathbf{r}^{n}$; $\mathbf{r}$.

6. *Crash*: We can also take X=∅ in part 3.This gives an update **0** such that **S(0)** is the empty model for each S, and $\mathbf{0}_S$ is the empty relation.

A New Operation: Synamic Modalities for Updates.

If φ is an epistemic proposition and **r** an update, then $[\mathbf{r}]\varphi$ is an epistemic proposition defined by

$$([\mathbf{r}]\varphi)_{\mathbf{S}} = \{s \in S : \text{ if } s\mathbf{r}_{\mathbf{S}}t, \text{ then } t \in \varphi_{\mathbf{S(r)}}\}$$

We shall presents a number of logical systems which contain *epistemic operators* of various types. These operators are closely related to aspects of the scenarios represented before:

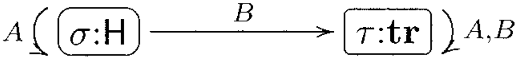
- The Logic of Public Announcements. $[\text{Pub }\varphi]\psi$.
- The Logic of Completely Private Announcements to Groups. $[\text{Pri}^{\mathcal{B}}\varphi]\psi$
- The Logic of Common Knowledge of Alternatives. $[\mathrm{Cka}^{\mathcal{B}}\vec{\varphi}]\psi$
  example (in Senario 3, S_1):
  $x \models \neg\Box_A H \wedge \langle Cka^{\{A\}} H, T\rangle(\Box_A(H \wedge \neg\Box_B\Box_A H) \wedge \Box_B(\Box_A H \vee \Box_A T))$
- The Logic of All Possible Epistemic Actions. $[\alpha]\varphi$

The Update Product Operation

In this section, we present the centerpiece of the formulation of our logical systems by introducing *action models, program models*, and an *update product operation*.

Epistemic Action Models:
Let Φ be the collection of all epistemic propositions. An *epistemic action model* is a triple $\Sigma = (\Sigma, \stackrel{\mathcal{A}}{\to}$,pre),where Σ is a set of *simple actions*, $\stackrel{\mathcal{A}}{\to}$ is an $\mathcal{A}$-indexed family of relations on Σ, and pre:$\Sigma \to \Phi$(the collection of all epistemic propositions).

Here comes an example – a completely private announcement to A that the coin is lying heads up:

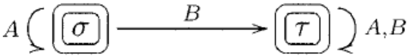


Formally, $\Sigma = \{\sigma, \tau\}$; $\sigma \xrightarrow{A} \sigma, \sigma \xrightarrow{B} \tau, \tau \xrightarrow{A} \tau, \tau \xrightarrow{B} \tau$; pre($\sigma$)= H, and pre($\tau$)= **tr**.

To model non-deterministic actions and non-simple actions,we define epistemic program models.

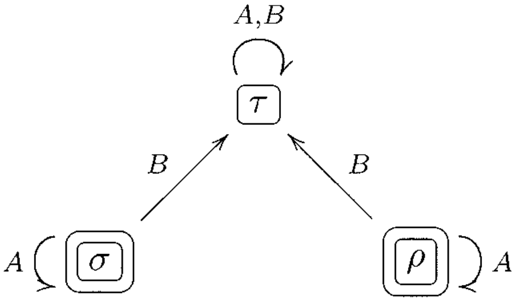
An epistemic program model is defined as $(\Sigma, \stackrel{\mathcal{A}}{\rightarrow}, pre, \Gamma)$,where Γ is a set of *designated simple actions*. When drawing the diagrams, we use doubled circles to indicate the designated actions in the set Γ.

Example. A Non-deterministic Action. *Either making a completely private announcement to A that the coin is lying heads up, or not making any announcement.*



Γ={ σ,τ }, pre(σ)=H, pre(τ)=**tr**

Example. A Determinstic, but Non-simple Action. *Completely privately announcing to A whether the coin is lying heads up or not*.



with pre(σ) = H, pre(τ) = **tr**, and pre(ρ) = ¬ H.

The Update Product of a State Model with an Epistemic Action Model:
Given a state model **S**=(S,$\stackrel{\mathcal{A}}{\rightarrow}_s$,$\| \cdot \|_S$) and an action model $\Sigma = (\Sigma, \stackrel{\mathcal{A}}{\rightarrow}$,pre), we define their *update product* to be the state model

$$\mathbf{S}\otimes\Sigma = (\mathbf{S}\otimes\Sigma, \stackrel{\mathcal{A}}{\rightarrow}, \| \cdot \|)$$

$$\mathbf{S}\otimes\Sigma = \{(s,\sigma) \in S\times : s \in pre(\sigma)_S\}$$

$$(s,\sigma) \stackrel{A}{\rightarrow} (s',\sigma') \text{ iff } s \stackrel{A}{\rightarrow} s' \text{ and } \sigma \stackrel{A}{\rightarrow} \sigma'$$

$$\| p \|_{\mathbf{S}\otimes\Sigma}$$

A program model induces an update.

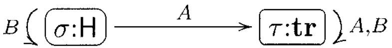
DEFINITION. Let (Σ, Γ) be a program model. We define an update which also denote (Σ, Γ) as follows:

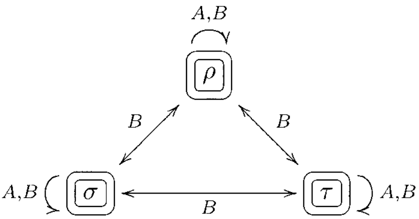
$$1. \mathbf{S}(\Sigma, \Gamma) = \mathbf{S} \otimes \Sigma$$

$$2. s(\Sigma, \Gamma)_{\mathbf{S}}(t, \sigma) \text{ iff } s = t \text{ and } \sigma \in \Gamma$$

Bisimulation Preservation.

Here comes SCENARIO 5 and 7 as examples of the Update Product:





Operations on Program Models:

- **1** and **0**.
- Sequential Composition. $\Sigma;\Delta = (\Sigma \times \Delta, \stackrel{\mathcal{A}}{\to}, pre_{\Sigma;\Delta}, \Gamma_{\Sigma;\Delta})$

  $(\sigma, \delta) \stackrel{A}{\to} (\sigma', \delta')$ iff $\sigma \stackrel{A}{\to} \sigma'$ and $\delta \stackrel{A}{\to} \delta'$

  $pre_{\Sigma;\Delta}(\sigma, \delta) = \langle(\Sigma, \sigma)\rangle pre_{\Delta}(\delta)$
- Disjoint Union. $\bigsqcup_{i\in I} \Sigma_i = (\bigsqcup_{i\in I} \Sigma_i, \stackrel{\mathcal{A}}{\to}, pre, \Gamma)$

  $\bigsqcup_{i\in I} \Sigma_i$ is $\bigcup_{i\in I}(\Sigma_i \times \{i\})$

  $(\sigma, i) \stackrel{\mathcal{A}}{\to} (\tau, j)$ iff $i = j$ and $\sigma \stackrel{\mathcal{A}}{\to}_i \tau$

- Iteration.

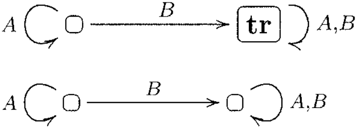
  $\Sigma^* = \bigsqcup\{\Sigma^n : n \in \mathbb{N}\}$. Here $\Sigma^0 = \mathbf{1}$, and $\Sigma^{n+1} = \Sigma^n; \Sigma$

PROPOSITION. The update induced by a composition of program models is the composition of the induced updates. Similarly for sums and iteration.

Logical Languages Based on Action Signatures

Now we introduce *logical languages based on action signatures*.

Action Signature: The notion of an action signature is an abstraction of the notion of action model.



an *enumeration without repetition*

DEFINITION. An *action signature* is a structure

$$\Sigma = (\Sigma, \overset{\mathcal{A}}{\to}, (\sigma_1, \sigma_2, \ldots, \sigma_n))$$

$\sigma_1, \sigma_2, \ldots, \sigma_n$ is a designated *listing* of a subset of Σ without repetitions. We call the elements of Σ *action types*, and the ones in the listing $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ *non-trivial action types*.

Two examples: Σ_{pub} and $Cka_k^{\mathcal{B}}$

We can use the abstract notion – the *action signature* – to regain *program models*: $(\Sigma, \Gamma)(\psi_1, \ldots, \psi_n)$.

For $j = 1, \ldots, n, pre(\sigma_j) = \psi_j$. $pre(\sigma) =$**tr** for all the other trivial actions.

To summarize: *every action signature, set of distinguished action types in it, and corresponding tuple of epistemic propositions gives an epistemic program model*.

Fix an action signature Σ. We can present a logic $\mathcal{L}(\Sigma)$:

sentences ϕ
true | p_i | $\neg\varphi$ | $\varphi \wedge \psi$ | $\Box_A \varphi$ | $\Box^*_{\mathcal{B}} \varphi$ | $[\pi]\varphi$

programs π
skip | crash | $\sigma\psi_1, \ldots, \psi_n$ | $\pi \sqcup \rho$ | $\pi; \rho$ | π^*

semantics
$[[[\pi]\varphi]] = [[[[\pi]]]][[\varphi]]$
$[[\sigma\psi_1, \ldots, \psi_n]] = (\Sigma, \sigma, [[\psi_1]], \ldots, [[\psi_n]])$

We require $\sigma \in \Sigma$. The actions of the form $\sigma_i\psi_1, \ldots, \psi_n (i \leqslant n)$ are called *non-trivial*. $(\Sigma, \sigma, [[\psi_1]], \ldots, [[\psi_n]])$ is a signature-based program model.

We generalize now our signature logics $\mathcal{L}(\Sigma)$ to families $\mathcal{S}$ of signatures – combine all the logics $\{\mathcal{L}(\Sigma)\}_{\Sigma\in\mathcal{S}}$ into a single lgic:

$\sigma\psi_1, \ldots, \psi_n$

where $\sigma \in \Sigma$, for some arbitrary signature $\Sigma \in \mathcal{S}$, and n is the length of the listing of non-trivial action types of Σ.
The semantics of $\sigma\psi_1, \ldots, \psi_n$ refers to the appropriate signature.

Example. Let $\mathcal{S} = \{\Sigma : \Sigma \text{ is a finite signature}\}$. The logic $\mathcal{L}(\mathcal{S})$ will be called the logic of all epistemic programs.

Preservation of Bisimulation and Atomic Propositions

Now we can formalize the several target logics as epistemic program logics $\mathcal{L}(\mathcal{S})$.

- *The Logic of Public Announcements* $\mathcal{L}(\Sigma_{Pub})$.

  $\mathbf{S}([[Pub\varphi]])$
  $= \mathbf{S}(\Sigma_{Pub}, Pub, [[\varphi]]) = \mathbf{S} \otimes (\Sigma_{Pub}, Pub, [[\varphi]])$
  $= \{(s, Pub) : s \in [[\varphi]]_{\mathbf{S}}\}$

- *Test-only PDL*.
  We have sentences of $[?\varphi]\chi$ and $[\mathrm{skip}\ \varphi]\chi$

$\mathbf{S}([[?\varphi]])$
$= \mathbf{S}(\Sigma_?, ?, [[\varphi]]) = \mathbf{S} \otimes (\Sigma_?, ?, [[\varphi]])$
$= \{(s, ?) : s \in [[\varphi]]_{\mathbf{S}}\} \cup \{(s, skip) : s \in S\}$

$[[[\text{skip } \varphi]\psi]]_{\mathbf{S}}$
$= \{s \in S : \text{if } s\ [[\text{skip } \varphi]]_{\mathbf{S}}\ t, \text{ then } t \in [[\psi]]_{\mathbf{S}([[\text{skip } \varphi]])}\}$
$= \{s \in S : (s, skip) \in [[\psi]]_{\mathbf{S}([[\text{skip } \varphi]])}\}$
$= \{s \in S : s \in [[\psi]]_{\mathbf{S}}\}$

That is, $[[[skip \varphi]\psi]]_{\mathbf{S}} = [[\psi]]_{\mathbf{S}}$

- *The Logic of Totally Private Announcements* $\mathcal{L}$(Pri).

  Pri = {Pri$^{\mathcal{B}}$: $\emptyset \neq \mathcal{B} \subseteq \mathcal{A}$}

  For example, in the case of $\mathcal{A} = \{A, B\}$, $\mathcal{L}$(Pri) will have basic actions of the forms: Pri$^{\mathrm{A}}$ φ, Pri$^{\mathrm{B}}$ φ, Pri$^{\mathrm{A,B}}$ φ, skip$^{\mathrm{A}}$ φ, skip$^{\mathrm{B}}$ φ, skip$^{\mathrm{A,B}}$ φ.

- *The Logic of Common Knowledge of Alternatives* $\mathcal{L}$(Cka).

  Cka = {Cka$^{\mathcal{B}}_{\mathrm{k}}$: $\emptyset \neq \mathcal{B} \subseteq \mathcal{A}, 1 \leq k$}

- *Logics Based on Frame Conditions.*
- *Announcements by Particular Agents.*
- *Lying*

  $\Sigma^{A}_{\mathrm{Lie}} = \{\mathrm{Secret}^{A}, \mathrm{Pub}^{A}\}$

  $\mathcal{L}(\Sigma^{A}_{\mathrm{Lie}})$ contains sentences like $[\mathrm{Secret}^{A}\,\varphi, \psi]\chi$.
- *Wiretapping, Paranoia etc.*

Endnote. This section is the cenerpiece of the paper, and all of the work in it is new.

Logical Systems

Now we present a *sound* proof system for the validities in $\mathcal{L}(\mathcal{S})$, and *sound* and *complete* proof system for $\mathcal{L}_1(\mathcal{S})$ amd $\mathcal{L}_0(\mathcal{S})$:

	Basic Axioms	
	All sentential validities	
	($[\pi]$-normality)	$\vdash [\pi](\varphi \to \psi) \to ([\pi]\varphi \to [\pi]\psi)$
	($\Box_A$-normality)	$\vdash \Box_A(\varphi \to \psi) \to (\Box_A\varphi \to \Box_A\psi)$
*	($\Box^*_{\mathcal{C}}$-normality)	$\vdash \Box^*_{\mathcal{C}}(\varphi \to \psi) \to (\Box^*_{\mathcal{C}}\varphi \to \Box^*_{\mathcal{C}}\psi)$
	Action Axioms	For basic actions σ only:
	(Atomic Permanence)	$\vdash [\sigma]p \leftrightarrow (\mathrm{PRE}(\sigma) \to p)$
	(Partial Functionality)	$\vdash [\sigma]\neg\chi \leftrightarrow (\mathrm{PRE}(\sigma) \to \neg[\sigma]\chi)$
	(Action-Knowledge)	$\vdash [\sigma]\Box_A\varphi \leftrightarrow (\mathrm{PRE}(\sigma) \to \bigwedge\{\Box_A[\sigma']\varphi : \sigma \xrightarrow{A} \sigma' \text{ in } \Omega\})$
**	**Action Mix Axiom**	$\vdash [\pi^*]\varphi \to \varphi \wedge [\pi][\pi^*]\varphi$
*	**Epistemic Mix Axiom**	$\vdash \Box^*_{\mathcal{C}}\varphi \to \varphi \wedge \bigwedge\{\Box_A\Box^*_{\mathcal{C}}\varphi : A \in \mathcal{C}\}$
	Skip Axiom	$\vdash [skip]\varphi \leftrightarrow \varphi$
	Crash Axiom	$\vdash [crash]\mathsf{false}$
	Composition Axiom	$\vdash [\pi;\rho]\varphi \leftrightarrow [\pi][\rho]\varphi$
	Choice Axiom	$\vdash [\pi \sqcup \rho]\varphi \leftrightarrow [\pi]\varphi \wedge [\rho]\varphi$

Modal Rules

	(Modus Ponens)	From $\vdash \varphi$ and $\vdash \varphi \to \psi$, infer $\vdash \psi$
	($[\pi]$-necessitation)	From $\vdash \psi$, infer $\vdash [\pi]\psi$
	($\Box_A$-necessitation)	From $\vdash \varphi$, infer $\vdash \Box_A \varphi$
*	($\Box^*_{\mathcal{C}}$-necessitation)	From $\vdash \varphi$, infer $\vdash \Box^*_{\mathcal{C}} \varphi$
**	**Program Induction Rule**	From $\vdash \chi \to \psi \wedge [\pi]\chi$, infer $\vdash \chi \to [\pi^*]\psi$
*	**Action Rule**	

Let ψ be a sentence, let α be a simple action, and let $\mathcal{C}$ be a set of agents. Let there be sentences χ_β for all β such that $\alpha \to^*_{\mathcal{C}} \beta$ (including α itself), and such that

1. $\vdash \chi_\beta \to [\beta]\psi$.
2. If $A \in \mathcal{C}$ and $\beta \xrightarrow{A} \gamma$, then $\vdash (\chi_\beta \wedge \mathrm{PRE}(\beta)) \to \Box_A \chi_\gamma$.

From these assumptions, infer $\vdash \chi_\alpha \to [\alpha]\Box^*_{\mathcal{C}}\psi$.

And we have *some derivable principles* from the proof system:

$$\vdash [\alpha]\Box^*_{\mathcal{C}} \to [\alpha]\psi$$

$$\text{From } \vdash \chi \to \psi \wedge \Box_{\mathrm{A}}\chi \text{ for all A, infer } \vdash \chi \to \Box^*_{\mathcal{A}}\psi.$$

Here we spell out what the axioms of $\mathcal{L}_1(\mathcal{S})$ come to when we specilize the general logic to *the target logics*.
The main points of the logic of public announcements:

Basic Axioms	
($[Pub\ \varphi]$-normality)	$\vdash [Pub\ \varphi](\psi \rightarrow \chi) \rightarrow ([Pub\ \varphi]\psi \rightarrow [Pub\ \varphi]\chi)$
Announcement Axioms	
(Atomic Permanence)	$\vdash [Pub\ \varphi]p \leftrightarrow (\varphi \rightarrow p)$
(Partial Functionality)	$\vdash [Pub\ \varphi]\neg\psi \leftrightarrow (\varphi \rightarrow \neg[Pub\ \varphi]\psi)$
(Announcement-Knowledge)	$\vdash [Pub\ \varphi]\Box_A\psi \leftrightarrow (\varphi \rightarrow \Box_A[Pub\ \varphi]\psi)$
Modal Rules	
($[Pub\ \varphi]$-necessitation)	From $\vdash \psi$, infer $\vdash [Pub\ \varphi]\psi$
Announcement Rule	From $\vdash \chi \rightarrow [Pub\ \varphi]\psi$ and $\vdash \chi \wedge \varphi \rightarrow \Box_A\chi$ for all A,infer $\vdash \chi \rightarrow [Pub\ \varphi]\Box^*_{\mathcal{A}}\psi$

As for the logic of completely private announcements to groups, the Action-Knowledge Axiom splits into two axioms:

$$[\mathrm{Pri}^{\mathcal{B}}\varphi]\Box_{\mathrm{A}}\psi \leftrightarrow (\varphi \to \Box_A[\mathrm{Pri}^{\mathcal{B}}\varphi]\psi) \text{ for A} \in \mathcal{B}$$

$$[\mathrm{Pri}^{\mathcal{B}}\varphi]\Box_{\mathrm{A}}\psi \leftrightarrow (\varphi \to \Box_A\psi) \text{ for A} \notin \mathcal{B}$$

As for the logic of common knowledge of alternatives. The Action-knowledge becomes:

$$[\mathrm{Cka}^{\mathcal{B}}\vec{\varphi}]\Box_{\mathrm{A}}\psi \leftrightarrow (\varphi_1 \to \Box_A[\mathrm{Cka}^{\mathcal{B}}\vec{\varphi}]\psi) \text{ for A} \in \mathcal{B}$$

$$[\mathrm{Cka}^{\mathcal{B}}\vec{\varphi}]\Box_{\mathrm{A}}\psi \leftrightarrow (\varphi_1 \to \bigwedge_{0 \le i \le k} \Box_A[\mathrm{Cka}^{\mathcal{B}}\vec{\varphi^i}]\psi) \text{ for A} \in \mathcal{B}$$

Now we give some examples in the target logics:

- $\vdash \Box^*_{\text{A, B}}(\text{H} \leftrightarrow \neg\text{T}) \rightarrow [\text{Pub H}]\Box^*_{\text{A,B}}\neg\text{T}$
- (What Happens when a Publicly Known Fact is Announced)
  $\Box^*\varphi \rightarrow ([\text{Pub}\varphi]\psi \leftrightarrow \psi)$
- (A Commutativity Principle for Private Announcements)
  $\vdash [\text{Pri}^{\mathcal{B}}\varphi_1][\text{Pri}^{\mathcal{C}}\varphi_2]\psi \leftrightarrow [\text{Pri}^{\mathcal{C}}\varphi_2][\text{Pri}^{\mathcal{B}}\varphi_1]\psi$
- (Actions Do Not Change Common Knowledge of Non-epistemic Sentences)
  $\vdash \psi \leftrightarrow [\alpha]\psi$
  $\vdash \Box^*_{\mathcal{C}}\psi \leftrightarrow [\alpha]\Box^*_{\mathcal{C}}\psi$

Conclusion

- This paper has shown how to define and study logical languages that contain constructs corresponding to epistemic acions.
- The key steps are the recognition that we can associate to a social action α a mathematical model Σ, i.e., a program model. It has features in common with the state models.
- The operation of update product enables one to build complex and interesting state models.
- The formalization of the target languages involve the signature-based languages $\mathcal{L}(\Sigma)$. These languages are needed to formulate the logic of private announcements, for example.
- many other problems to be explained...

Thank you for your attention!